4 pages

40/40

UNIVERSITY OF REGINA
DEPARTMENT OF MATHEMATICS & STATISTICS

STAT 289 - STATISTICS & NUMERICAL ANALYSIS FOR ENGINEERS

TEST #2

Thursday, Aug. 9, 2001

1. A complex electronic system is built with a certain number of backup components in its subsystems. One subsystem has four identical components, each with a probability of .2 of failing in less than 1000 hours. The subsystem will operate if any two of the four components are operating. Assume that the components operate independently.

   (a) Find the probability that exactly two of the four components last longer than 1000 hours. Let X = the no. of operating [illegible]

   p = probability of 1 component operating > 1000 hrs
   q = probability of 1 component [illegible]
   p = 1 - .2 = .8 n = 4 q = 1 - p = 1 - .8 = .2 ✓
   P(X = 2) = [illegible] (.2) = .1536

[8]

8

   (b) Find the probability that the subsystem operates longer than 1000 hours.

   P(X ≥ 2) = 1 - P(X < 2) = 1 - [illegible]
   P(1) = [illegible] (.8)(.2) [illegible] = .0256 ✓
   P(0) = [illegible] = .0016
   1 - [illegible] = .9728

2. There are two entrances to a parking lot. Cars arrive at entrance I according to a Poisson distribution at an average of three per hour, and at the entrance II according to a Poisson distribution at an average of four per hour. What is the probability that three cars arrive at a parking lot in a given hour? (Assume that the numbers of cars arriving at the two entrances are independent).

[6]

6

3. The time for an automated system in a warehouse to locate a part is normally distributed with a mean of 45 seconds and a standard deviation of 30 seconds. Suppose that independent requests are made for 10 parts. What is the probability that the average time to locate 10 parts exceed 60 seconds?

[6]

6

4. The length of time required by students to complete a 1-hour exam is a continuous random variable with density function given by

[10]

$$f(y) = \begin{cases} cy^2 + y, & 0 \le y \le 1 \\ 0, & elsewhere \end{cases}$$

(a) Find the value of c that makes $f(y)$ a probability density function.

$\boxed{c = \frac{1}{2}}$

(b) Find the probability that a student finishes in less than half hour.

10

(c) Find the expected value and variance of this random variable.

5. Scores on an examination are assumed to be normally distributed with a mean of 78 and variance of 36. $\mu = 78 \quad \sigma^2 = 36 \quad \sigma = 6$

[10]

(a) What is the probability that a person taking the examination scores higher than 72?

$P(X > 72) = 1 - P(X \le 72)$

$= 1 - P(Z \le -1) = 1 - \Phi(-1) = 1 - .158655 = \boxed{.841345}$

(b) Suppose that students scoring in the top 10% of this distribution are to receive an A grade. What is the minimum score a student must achieve to earn an A grade?

$P(X > x_0) = .10 \qquad P(X \le x_0) = 1 - .10 = .9$

$\therefore x_0 = 85.74$

10

(c) What must be the cutoff point for passing the examination if the examiner wants only the top 28.1% of all scores to be passing?

$P(X \ge x_0) = .281$

$P(Z \le \ldots) = .719 \qquad \therefore z = .58$

$\therefore x_0 = 81.48$