

UNIVERSITY OF REGINA
DEPARTMENT OF MATHEMATICS & STATISTICS



STAT 289 - STATISTICS & NUMERICAL ANALYSIS FOR ENGINEERS

TEST #2

Thursday, Aug. 9, 2001

1. A complex electronic system is built with a certain number of backup components in its subsystems. One subsystem has four identical components, each with a probability of .2 of failing in less than 1000 hours. The subsystem will operate if any two of the four components are operating. Assume that the components operate independently.

- (a) Find the probability that exactly two of the four components last longer than 1000 hours.

Let $X =$ the no. of operating components
 $p =$ probability of 1 component operating > 1000 hrs
 $q =$ probability of 1 component failing < 1000 hrs
 $p = 1 - 0.2 = 0.8$ $n = 4$ $q = 1 - p = 1 - 0.8 = 0.2$ ✓
 $P(X = 2) = \binom{4}{2} (0.8)^2 (0.2)^2 = 0.1536$

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- (b) Find the probability that the subsystem operates longer than 1000 hours.

$P(X \geq 2) = 1 - P(X < 2) = 1 - (P(X=0) + P(X=1))$
 $P(X=0) = \binom{4}{0} (0.8)^0 (0.2)^4 = 0.0016$
 $P(X=1) = \binom{4}{1} (0.8)^1 (0.2)^3 = 0.0256$ ✓
 $\therefore 1 - (0.0016 + 0.0256) = 0.9728$



2. There are two entrances to a parking lot. Cars arrive at entrance I according to a Poisson distribution at an average of three per hour, and at the entrance II according to a Poisson distribution at an average of four per hour. What is the probability that three cars arrive at a parking lot in a given hour? (Assume that the numbers of cars arriving at the two entrances are independent).

$\lambda_1 = 3$
 $\lambda_2 = 4$
 $\lambda = 3 + 4 = 7$
 $P(X=3) = \frac{e^{-7} 7^3}{3!}$

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3. The time for an automated system in a warehouse to locate a part is normally distributed with a mean of 45 seconds and a standard deviation of 30 seconds. Suppose that independent requests are made for 10 parts. What is the probability that the average time to locate 10 parts exceed 60 seconds?

$P(\bar{X} > 60) = P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} > \frac{60 - 45}{30/\sqrt{10}}\right)$

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$= P(Z > 1.5811)$
 $= 1 - 0.942947 = 0.057053$

4. The length of time required by students to complete a 1-hour exam is a continuous random variable with density function given by

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$$f(y) = \begin{cases} cy^2 + y, & 0 \leq y \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

- (a) Find the value of c that makes $f(y)$ a probability density function.

Handwritten solution for (a):

$$\int_0^1 (cy^2 + y) dy = 1$$

$$\left[\frac{cy^3}{3} + \frac{y^2}{2} \right]_0^1 = 1$$

$$\frac{c}{3} + \frac{1}{2} = 1$$

$$\frac{c}{3} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$c = \frac{3}{2}$$

$c = \frac{3}{2}$ ✓

- (b) Find the probability that a student finishes in less than half hour.

Handwritten solution for (b):

$$P(Y < 0.5) = \int_0^{0.5} f(y) dy$$

$$= \int_0^{0.5} \left(\frac{3}{2}y^2 + y \right) dy$$

$$= \left[\frac{3}{2} \cdot \frac{y^3}{3} + \frac{y^2}{2} \right]_0^{0.5}$$

$$= \left[\frac{y^3}{2} + \frac{y^2}{2} \right]_0^{0.5}$$

$$= \frac{(0.5)^3}{2} + \frac{(0.5)^2}{2}$$

$$= \frac{0.125}{2} + \frac{0.25}{2}$$

$$= \frac{0.375}{2} = 0.1875$$

- (c) Find the expected value and variance of this random variable.

Handwritten solution for (c):

Expected value $E(Y)$:

$$E(Y) = \int_0^1 y f(y) dy = \int_0^1 y (cy^2 + y) dy$$

$$= \int_0^1 (cy^3 + y^2) dy$$

$$= \left[\frac{cy^4}{4} + \frac{y^3}{3} \right]_0^1$$

$$= \frac{c}{4} + \frac{1}{3}$$

$$= \frac{3}{2} \cdot \frac{1}{4} + \frac{1}{3} = \frac{3}{8} + \frac{1}{3} = \frac{9}{24} + \frac{8}{24} = \frac{17}{24}$$

Variance $V(Y)$:

$$V(Y) = E(Y^2) - [E(Y)]^2$$

$$E(Y^2) = \int_0^1 y^2 f(y) dy = \int_0^1 y^2 (cy^2 + y) dy$$

$$= \int_0^1 (cy^4 + y^3) dy$$

$$= \left[\frac{cy^5}{5} + \frac{y^4}{4} \right]_0^1$$

$$= \frac{c}{5} + \frac{1}{4}$$

$$= \frac{3}{2} \cdot \frac{1}{5} + \frac{1}{4} = \frac{3}{10} + \frac{1}{4} = \frac{6}{20} + \frac{5}{20} = \frac{11}{20}$$

$$V(Y) = \frac{11}{20} - \left(\frac{17}{24} \right)^2 = \frac{11}{20} - \frac{289}{576}$$

$$= \frac{11 \cdot 288}{20 \cdot 576} - \frac{289}{576} = \frac{3168}{11520} - \frac{289}{576}$$

$$= \frac{3168}{11520} - \frac{578}{11520} = \frac{2590}{11520} = \frac{259}{1152}$$

$V(Y) = \frac{259}{1152} \approx 0.2248$ ✓

5. Scores on an examination are assumed to be normally distributed with a mean of 78 and variance of 36. $\mu = 78$ $\sigma^2 = 36$ $\sigma = 6$

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- (a) What is the probability that a person taking the examination scores higher than 72?

$P(X > 72) = 1 - P(X \leq 72) = 1 - \Phi\left(\frac{72 - 78}{6}\right) = 1 - \Phi(-1) = 1 - 0.242038 = 0.757962$

≈ 0.758

- (b) Suppose that students scoring in the top 10% of this distribution are to receive an A grade. What is the minimum score a student must achieve to earn an A grade?

$P(X \geq x) = 0.10 \Rightarrow P(X \leq x) = 1 - 0.10 = 0.9$

$\Phi\left(\frac{x - 78}{6}\right) = 0.9 \Rightarrow \frac{x - 78}{6} = 1.28$

$x = 78 + 6(1.28) = 85.68$

- (c) What must be the cutoff point for passing the examination if the examiner wants only the top 28.1% of all scores to be passing?

$P(X \geq x) = 0.281 \Rightarrow P(X \leq x) = 1 - 0.281 = 0.719$

$\Phi\left(\frac{x - 78}{6}\right) = 0.719 \Rightarrow \frac{x - 78}{6} = 0.58$

$x = 78 + 6(0.58) = 81.48$