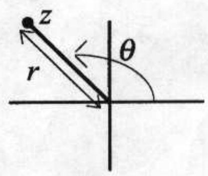
IEEE

Math 215

Prof. J. Chris Fisher — **Final Exam 97F** — December 12, 1997

Marks

20 1. For all parts to this question, z is a point in the second quadrant; its polar coordinates are (r, θ) where r is a fixed positive number and θ is an angle for which $\sin\theta = 0.6$.



a. Write z in the form $a + \boldsymbol{i}\,b$.
b. Write e^z in the form $a + \boldsymbol{i}\,b$.
c. Write $\operatorname{Ln} z$ in the form $a + \boldsymbol{i}\,b$.

15 2. One of the following two functions is analytic everywhere, the other is analytic nowhere.

$$f(z) = 3x - 2xy + \boldsymbol{i}(y^2 - x)$$
$$g(z) = 3x - 2xy + \boldsymbol{i}(x^2 + 3y - y^2)$$

a. Explain which is which and why.
b. (Recall that nonanalytic functions might have derivatives at isolated points.) At what point does the nonanalytic function have a derivative? What is that derivative (written in the form $\ldots + \ldots \boldsymbol{i}$)?

3. For both parts of this question $A = \begin{bmatrix} -10 & -11 & 5 \\ 7 & 8 & -3 \\ -4 & -4 & 3 \end{bmatrix}$. You can take my word for it that $\det(A - \lambda I) = -(-1+\lambda)^2(1+\lambda)$.

10 a. Find all of the eigenvalues and eigenvectors of A. (Show the main steps in finding the eigenvectors by hand, even if you use a calculator. No credit will be given if the derivation does not appear.)

15 b. Find the unique solution to the system $\begin{cases} \dfrac{dx}{dt} = -10x - 11y + 5z \\ \dfrac{dy}{dt} = 7x + 8y - 3z \\ \dfrac{dz}{dt} = -4x - 4y + 3z \end{cases}$

for which $x(0) = 3$, $y(0) = 4$, $z(0) = 5$.

20 4. All parts of this question refer to the function

$$f(x) = \sin x,\ 0 \le x \le \pi/2.$$

a. Find a_{1997} and b_{1997} for *each* of f(x) and c(x) (4 coefficients in all) where f(x) and c(x) are, respectively, the Fourier and the cosine expansions.

Selected integrals:

$$\int \sin ax \sin bx\,dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)},\qquad \text{for } a \ne \pm b$$
$$\int \sin ax \cos bx\,dx = -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)}$$

b. Find $f(5\pi/6)$, $c(5\pi/6)$, and $s(5\pi/6)$ (the values of the Fourier, the cosine, and the sine expansions at $x = 5\pi/6$).

20 5. All parts of this question refer to the series solution $y = c_0 + c_1x + c_2x^2 + \ldots$ of the differential equation

$$y'' + 4y = 6x + 2.$$

a. For what values of x is the power series solution (about $x = 0$) guaranteed to exist. Explain.

b. Find a formula for the coefficient c_n in terms of the previous c_k (with $k < n$) that holds for all $n \ge 2$.

c. Find the values of c_4 and c_5 when y is the unique solution that satisfies the initial conditions

$$y(0) = 1,\quad y'(0) = 3.$$