W 2000

2 pages

$\frac{15}{15}$

UNIVERSITY OF REGINA
DEPARTMENT OF MATHEMATICS & STATISTICS
Mathematics 213-001
Quiz No. 2
200010
W 2000

Time: 20 minutes
Instructor: Dr. S.K. Kaul

NAME:
STUDENT NO.:

1. Given the parametric equation of the curve **C**:

$$\mathbf{F}(t) = t^3\mathbf{i} + t^3\mathbf{j} + t^3\mathbf{k}, \quad -1 \le t \le 1$$

(a) Find a length funtion $s = s(t)$ for **C**.

$F(u) = u^3 i + u^3 j + u^3 k$

$F'(u) = 3u^2 i + 3u^2 j + 3u^2 k$

$\|F'(u)\| = \sqrt{3(9u^4)}$

$= \sqrt{27u^4} = 3u^2\sqrt{3}$

$s(t) = \int_{-1}^{t} \|F'(u)\| \, du$

$s(t) = \sqrt{3} \int_{-1}^{t} 3u^2 \, du$

$= \sqrt{3} \left(\frac{3}{3}u^3\right) \left[u \Big|_{-1}^{t} \right]$

$= \sqrt{3}\, u^3 \Big|_{-1}^{t}$

$s(t) = \sqrt{3}(t^3 + 1)$

(b) Write the position vector of **C** as a function of s.

$s(t) = \sqrt{3}(t^3+1)$

$t = \sqrt[3]{\frac{s}{\sqrt{3}} - 1}$

$G(s) = F(t(s)) = \left(\sqrt[3]{\frac{s}{\sqrt{3}} - 1}\right)^3 i + \left(\sqrt[3]{\frac{s}{\sqrt{3}} - 1}\right)^3 j + \left(\sqrt[3]{\frac{s}{\sqrt{3}} - 1}\right) k$

$= \left(\frac{s}{\sqrt{3}} - 1\right) i + \left(\frac{s}{\sqrt{3}} - 1\right) j + \left(\frac{s}{\sqrt{3}} - 1\right) k$

(c) Verity that the resulting position vector has a derivative of length 1.

$G'(s) = \frac{1}{\sqrt{3}} i + \frac{1}{\sqrt{3}} j + \frac{1}{\sqrt{3}} k$

$\|G'(s)\| = \sqrt{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}}$

2. (a) Find the unit tangent ($\mathbf{T}$), unit normal ($\mathbf{N}$) for the curve $\mathbf{C}$:

$$\mathbf{F}(t) = \sin 4t\mathbf{i} + 3t\mathbf{j} + \cos 4t\mathbf{k}. \tag{1}$$

$F'(t) = 4\cos 4t\, i + 3j - 4\sin 4t\, k$

$\|F'(t)\| = \sqrt{(4\cos 4t)^2 + (3)^2 + (4\sin 4t)^2}$

$= \sqrt{16(\cos^2 4t + \sin^2 4t) + 9}$

$= \sqrt{16+9}$

$= \sqrt{25} = 5$

$T = \dfrac{F'(t)}{\|F'(t)\|} = \frac{1}{5}(4\cos 4t\, i + 3j - 4\sin 4t\, k)$

$= \frac{4}{5}\cos 4t\, i + \frac{3}{5} j - \frac{4}{5}\sin 4t\, k$

$T' = -\frac{16}{5}\sin 4t\, i - \frac{16}{5}\cos 4t\, k$

$\|T'\| = \sqrt{(\frac{16}{5})^2(\sin^2 4t + \cos^2 4t)}$

$= \sqrt{(\frac{16}{5})^2} = \frac{16}{5}$

$N = \dfrac{T'}{\|T'\|}$

$= \frac{5}{16}(-\frac{16}{5}\sin 4t\, i - \frac{16}{5}\cos 4t\, k)$

$= -\sin 4t\, i - \cos 4t\, k$

or $\kappa = \dfrac{\|T'\|}{\|F'\|} = \dfrac{16/5}{5} = \frac{16}{25}$

(b) Find the curvature of the curve $\mathbf{C}$ in (1).

$\kappa = \dfrac{\|F'(t) \times F''(t)\|}{\|F'(t)\|^3}$

$F'(t) = 4\cos 4t\, i + 3j - 4\sin 4t\, k$

$F''(t) = -16\sin 4t\, i - 16\cos 4t\, k$

16 × 3 = 48 16 × 4 = 64

$F' \times F'' = \begin{vmatrix} i & j & k \\ 4\cos 4t & 3 & -4\sin 4t \\ -16\sin 4t & 0 & -16\cos 4t \end{vmatrix}$

$= i(-48\cos 4t) - j(-64\cos^2 4t - 64\sin^2 4t) + k(48\sin 4t)$

$= (-48\cos 4t)i + 64j + (48\sin 4t)k$

$\sqrt{48^2(\cos^2 4t + \sin^2 4t) + 64^2}$

$= \dfrac{\sqrt{48^2 + 64^2}}{(5)^3}$

$= \dfrac{80}{125}$

$= \dfrac{16}{25}$