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UNIVERSITY OF REGINA
DEPARTMENT OF MATHEMATICS & STATISTICS

Mathematics 212 - 001

Midterm Examination

October, 24

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Time: 75 min

Name: _____

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Student No.: _____

1. Sequences. (5)+(5)

a) Give the general term of the sequence

$$\frac{1}{11}, \frac{2}{101}, \frac{3}{1001}, \dots$$

$$a_n = \frac{n}{1+10^n}$$

✓

b) Find a formula for the general term a_n of the sequence

$$1, \frac{1 \cdot 2 \cdot 3}{4}, \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}{16}, \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7}{64}, \dots$$

$$a_n = \frac{(2n-1)!}{4^{n-1}}$$

✓

2. In the following determine whether the given series is convergent or divergent. (10)+(10)+(10)

a) $\sum_{n=1}^{\infty} \frac{2^n}{n^{10}}$

b) $\sum_{n=1}^{\infty} \frac{1}{(n+1)\ln(n+1)}$

c) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+1}$

a) by ratio test
 $\lim_{n \rightarrow \infty} \left| \frac{2^{n+1}}{(n+1)^{10}} \cdot \frac{n^{10}}{2^n} \right|$

$\lim_{n \rightarrow \infty} \frac{2n^{10}}{n^{10}}$

$\lim_{n \rightarrow \infty} 2$

$\lim_{n \rightarrow \infty} \frac{2}{1 + \frac{1}{n} + \frac{1}{n^2} + \dots}$

$= 2$

2 > 1 so series diverges

b) $\ln(\)$

c) by alternating series test

$\lim_{n \rightarrow \infty} \frac{n}{n^2+1} = 0$

$\lim_{n \rightarrow \infty} \frac{n}{n^2+1} > 0$

$\lim_{n \rightarrow \infty} \frac{n}{n^2+1} < 0$

converges

3. Determine whether the given series is convergent or divergent. If it is convergent, find its sum. $(5)+(5)+(5)$
 (here i is imaginary unit).

a) $\sum_{n=0}^{\infty} \frac{1}{2^n}$ b) $\sum_{n=0}^{\infty} \frac{i}{3^n}$ c) $\sum_{n=0}^{\infty} (\frac{1}{2^n} + i \cdot \frac{1}{3^n})$

Handwritten notes for part a:
 a) ratio test
 $\lim_{n \rightarrow \infty} \frac{1/2^{n+1}}{1/2^n} = \frac{1}{2} < 1$
 convergent

Handwritten notes for part b:
 b) ratio test
 $\lim_{n \rightarrow \infty} \frac{i/3^{n+1}}{i/3^n} = \frac{1}{3} < 1$
 convergent

Handwritten notes for part c:
 $\sum_{n=0}^{\infty} \frac{1}{2^n} + i \sum_{n=0}^{\infty} \frac{1}{3^n}$
 $= 2 + i \cdot 1.5 = 2 + 1.5i$

~~*Handwritten work for part a:*
 $8 + \frac{1}{16} + \dots$
 $a_n = 1/2^n$
 $\lim_{n \rightarrow \infty} \frac{1/2^{n+1}}{1/2^n} = 1/2 < 1$
 sum = 2~~

~~*Handwritten work for part b:*
 $4 + \frac{1}{27} + \dots$
 $a_n = i/3^n$
 $\lim_{n \rightarrow \infty} \frac{i/3^{n+1}}{i/3^n} = 1/3 < 1$
 sum = 1.5i~~

$\frac{2 + 1.5i}{1 + i(\frac{1}{3})}$

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4. Find the radius of convergence and interval of convergence of the series (25)

$$\sum_{n=1}^{\infty} \frac{1}{n^2} (x-2)^n$$

Power series form: $\sum a_n (x-c)^n$
 $\therefore c = 2$

$$\lim_{n \rightarrow \infty} \left| \frac{x_{n+1}}{x_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(x-2)^{n+1}}{(n+1)^2} \cdot \frac{n^2}{(x-2)^n} \right|$$

$$= \lim_{n \rightarrow \infty} \left| \frac{(x-2)}{(1 + \frac{1}{n})^2} \right|$$

$$= |x-2| \lim_{n \rightarrow \infty} \left(\frac{n^2}{(n+1)^2} \right) = |x-2| \cdot 1$$

$$|x-2| < 1$$

$$-1 < x-2 < 1$$

$$1 < x < 3$$

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interval of convergence
 $[1 < x < 3]$

~~Interval of convergence
 $\sum_{n=1}^{\infty} \frac{1}{n^2} (x-2)^n$
 $\sum_{n=1}^{\infty} \frac{1}{n^2} (x-2)^n$
 $\sum_{n=1}^{\infty} \frac{1}{n^2} (x-2)^n$~~

5. Complex numbers. (5)+(15)

a) Determine which complex number is closer to the origin

$z = 10 + 8i$ or $z = 11 - 6i$

$$r_1 = \sqrt{10^2 + 8^2}$$

$$= \sqrt{100 + 64}$$

$$= \sqrt{164}$$

$$r_1 = 12.8$$

$$r_2 = \sqrt{11^2 + (-6)^2}$$

$$= \sqrt{121 + 36}$$

$$= \sqrt{157}$$

$$r_2 = 12.53$$

b) Find all solutions of the equation

$$z^5 = 2$$

$$z = \sqrt[5]{2}$$

$$z = \sqrt[5]{2} \left(\cos\left(\frac{0 + 2\pi k}{5}\right) + i \sin\left(\frac{0 + 2\pi k}{5}\right) \right)$$

$$k=0 \Rightarrow \sqrt[5]{2} \left(\cos\left(\frac{0}{5}\right) + i \sin\left(\frac{0}{5}\right) \right)$$

$$= \sqrt[5]{2} (1 + i(0))$$

$$= 1.149 + i(0)$$

Good luck!

$$k=1 \Rightarrow \sqrt[5]{2} \left(\cos\left(\frac{2\pi}{5}\right) + i \sin\left(\frac{2\pi}{5}\right) \right)$$

$$= 0.355 + i(1.092)$$

$$k=2 \Rightarrow \sqrt[5]{2} \left(\cos\left(\frac{4\pi}{5}\right) + i \sin\left(\frac{4\pi}{5}\right) \right)$$

$$= -0.329 + i(0.675)$$

$$k=3 \Rightarrow \sqrt[5]{2} \left(\cos\left(\frac{6\pi}{5}\right) + i \sin\left(\frac{6\pi}{5}\right) \right)$$

$$= -0.329 - i(0.675)$$

$$k=4 \Rightarrow \sqrt[5]{2} \left(\cos\left(\frac{8\pi}{5}\right) + i \sin\left(\frac{8\pi}{5}\right) \right)$$

$$= 0.355 - i(1.092)$$