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Math111

UNIVERSITY OF REGINA
DEPARTMENT OF MATHEMATICS & STATISTICS
Mathematics 111-001
Midterm 1
99 Fall

Time: 50 minutes
Instructor: J.H. Weston

NAME. _
STUDENT NO.: _

10 1. (a) Differentiate the function $f(x) = tan^{-1}(x^2 + e^{3x})$.

$f'(x) = \frac{1}{1+(x^2+e^{3x})^2} \cdot (2x + e^{3x}(3))$

$f'(x) = \frac{2x+3e^{3x}}{1+(x^2+e^{3x})^2}$

10 (b) Find the derivative of $f(x) = ln\frac{x^2-1}{x^3-1}$.

$= ln(x^2+1) - ln(x^3+1)$

$f'(x) = \frac{1}{x^2+1}(2x) - \ldots$

$\frac{x^3+1}{x^2+1}$

$= \frac{2x(x^3+1) - 3x^2(x^2+1)}{(x^2+1)(x^3+1)}$

20 2. Find any maxima and minima values of the function $f(x) = e^x - 2x$.

$f'(x) = 0$ for min

$f'(x) = e^x - 2 = 0$

$e^x = 2$ $\qquad \ln 2 = x \qquad \ln e = x$

$\ln e^x = \ln 2$ $\qquad e^x = e$

$x \ln e = \ln 2$

$x = \frac{\ln 2}{\ln e}$ $\qquad \frac{\ln 2}{1} = \ln 2$

$f''(x) = e^x$

$f''(\ln 2) = e^{(\ln 2)} = 2$ pos

$\therefore$ $f(\ln 2) = 2 - 2(\ln 2) = 2(1 - \ln 2) \cong .613$ is a minimum

$2(1 - \ln 2) \cong .6137$ is the minimum value

20 3. Find the equation of the tangent line to $y = e^{\cos x}$ at $x = \pi/2$. $y = 1$

$m = y' = e^{\cos x}(-\sin x) = -\sin x\, e^{\cos x}$

$m = f'(\frac{\pi}{2}) = -\sin(\frac{\pi}{2})e^{0} = -1 \cdot 1 = -1$

$y - y_1 = m(x - x_1)$

$y - 1 = -1(x - \frac{\pi}{2})$

$y - 1 = -x + \frac{\pi}{2}$

$y + x - 1 - \frac{\pi}{2} = 0 \Longrightarrow 2y + 2x - 2 - \pi = 0$

20 4. Evaluate

$$\lim_{x \to \infty} x^3 e^{-x^2}. \quad = \lim_{x \to \infty} \frac{x^3}{e^{x^2}} \quad \text{form } \frac{\infty}{\infty}$$

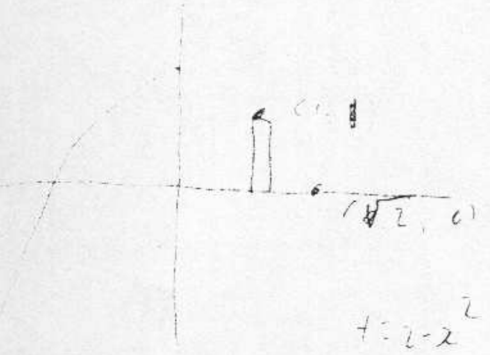
$$= \lim_{x \to \infty} \frac{3x^2}{e^{x^2}(2x)} = \lim_{x \to \infty} \frac{3x}{2e^{x^2}} = \lim_{x \to \infty} \frac{3}{\ldots} = \lim_{x \to \infty} \frac{3}{4} \cdot \lim_{x \to \infty} \ldots$$

$$\lim \frac{\infty}{\infty}$$

$$\frac{3}{4} \cdot (0) = \boxed{0}$$

20 5. Find the volume of the solid obtained by rotating, about the y-axis, the region bounded by
$x = 0, y = x$, and $y = 2 - x^2$.

$y = -x^2 + 2$



$x = 2 - x^2$

$x^2 - 2 + x = 0$

$x^2 + x - 2 = 0$

$(x - 1)(x + 2)$

$x = 1 \qquad x = -2$

$y = 2 - x^2$

$x^2 = 2 - y$

$x = \sqrt{2 - y}$

$x = 0$

$H = 2 - x^2$

$r = x$

$V = 2\pi r h \Delta x = 2\pi (x)(2 - x^2)\Delta x$
$= 2\pi(2x - x^3)\Delta x$

$V = 2\pi \int_0^{\sqrt{2}} 2x - x^3 \, dx$

$= 2\pi \left[x^2 - \frac{x^4}{4} \right] \Big|_0^{\sqrt{2}} = 2\pi \left[(\sqrt{2})^2 - \frac{(\sqrt{2})^4}{4} - 0 \right]$

$= 2\pi (2 - 1 - 0)$

$= 2\pi(1) = \boxed{2\pi \text{ units}^3}$