3 pages

$\frac{\cos^2 x}{\cos^2} + \frac{\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2}$

$1 + \tan^2 x = \sec^2 x$

$\frac{81}{20}$

UNIVERSITY OF REGINA

DEPARTMENT OF MATHEMATICS & STATISTICS

Mathematics 111-003

Midterm 2

Winter 2000

Time: 50 minutes

Instructor: J.H. Weston

20 1. Evaluate the following integrals.

(a) $\int \frac{\ln x}{\sqrt{x}} dx = \int \ln x \, x^{-\frac{1}{2}} dx$

$\int \ln x \, x^{-\frac{1}{2}} dx = 2 \ln x \sqrt{x} - 2\int \frac{\sqrt{x}}{x} dx$

$= 2\ln x\sqrt{x} - 2(2\sqrt{x}) + C = \boxed{2\sqrt{x}(\ln x - 2) + C}$

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(b) $\int \frac{dx}{x^2\sqrt{9-x^2}} = \int \frac{dx}{x^2 \, 3\sqrt{1-\left(\frac{x}{3}\right)^2}}$

$1-x^2 \quad \sin\theta$

use $x = 3\sin\theta$

$\frac{x}{3} = \sin\theta$

let $x = 3\sin\theta$

$dx = 3\cos\theta \, d\theta$

$= \int \frac{\cancel{3}\cos\theta \, d\theta}{9\sin^2\theta \, \cancel{3}\sqrt{1-\sin^2\theta}}$

$= \frac{1}{9}\int \frac{\cos\theta \, d\theta}{\sin^2\theta \, \cos\theta} = \frac{1}{9}\int \csc^2\theta \, d\theta$

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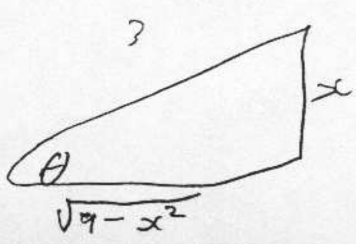
$= -\frac{1}{9}\cot\theta + C = \boxed{\frac{-\sqrt{9-x^2}}{81} + C}$

$\frac{-\sqrt{9-x^2}}{9x}$

$= b^2 + a^2$



$\therefore \cot\theta = \frac{\sqrt{9-x^2}}{\cancel{9}x}$

-1

note not difine at $x=0$

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(c) $\int_1^\infty \frac{1}{x(\ln x)^{3/2}} dx$

let $u = \ln x$ $du = \frac{1}{x} dx$

$\int \frac{1}{u^{\frac{3}{2}}} du = \int u^{-\frac{3}{2}} du$

$= -2u^{-\frac{1}{2}} = \frac{-2}{\sqrt{\ln x}}$

$\lim_{t\to\infty} \int_1^t \frac{1}{x(\ln x)^{\frac{3}{2}}} dx = \lim_{t\to\infty} \frac{-2}{\sqrt{\ln x}} \Big|_1^t$

$= \lim_{t\to\infty} \frac{-2}{\sqrt{\ln t}} - \frac{-2}{\sqrt{\ln 1}}$

$\therefore \int_1^\infty \frac{1}{x(\ln x)^{\frac{3}{2}}}$ is ~~converge~~ divergent

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(d) $\int \frac{dx}{x^2 - 2x + 3}$ $= \int \cancel{\frac{dx}{x(x - 2 + \frac{3}{x})}}$ ~~let $u = x - 2 + \frac{3}{x}$ $du = 1 +$~~

see $x^2-2x+3 = x^2-2x+1+2$
$= (x-1)^2+2$

$= \int \frac{dx}{(x-1)^2+2}$

$\tan^2+1$

~~$x-1 = \sqrt{2}\tan\theta$~~

$x-1 = \sqrt{2}\tan\theta$

$= \int \frac{dx}{2\tan^2\theta + 2}$

$dx = \sqrt{2}\sec^2\theta\, d\theta$

$\tan\theta = \frac{x-1}{\sqrt{2}}$

$= \frac{1}{2}\int \frac{\sqrt{2}\sec^2\theta\, d\theta}{(\tan^2\theta+1)}$

$\theta = \tan^{-1}\left(\frac{x-1}{\sqrt{2}}\right)$

$= \frac{1}{2}\int \sqrt{2}\, d\theta = \frac{\sqrt{2}}{2}\theta = \frac{\sqrt{2}}{2}\tan^{-1}\left(\frac{x-1}{\sqrt{2}}\right) + C$

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2. Are the following series convergent? Why?

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(a) $\sum_{n=1}^{\infty} \frac{1}{n+\ln n}$ $b_n = \cancel{\frac{1}{n} > \frac{1}{n+\ln n}}$ because $\frac{1}{n}$ small the $n+\ln n$

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$\sum_{n=1}^{\infty} \frac{1}{n}$ is the harmonic series It diverges

The comparison Test

if $b_n > a_n$ and b_n diverges then

same does a_n

$\therefore \sum_{n=1}^{\infty} \frac{1}{n+\ln n}$ diverges (is not convergent) comparison test

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(b) $\sum_{n=1}^{\infty} \frac{n}{e^n}$ Ratio test

$\lim_{n\to\infty} \frac{n+1}{e^{n+1}} \times \frac{e^n}{n} = \lim_{n\to\infty} \frac{n+1}{e\,n}$

always pos

$= \frac{1}{e} \lim_{n\to\infty} \frac{n+1}{n} = \frac{1}{e}\left[\frac{1+0}{1}\right] = \frac{1}{e}$

because the limit of $n\to\infty$ $\frac{a_{n+1}}{a_n} = \frac{1}{e} < 1$

then $\sum_{n=1}^{\infty} \frac{n}{e^n}$ converges

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