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Math 110-004: Calculus I

Lab Test # 5, Nov. 22, 1999.

Name:
Student number:

Let $f(x) = x^3 - 3x$. Find the intervals where f is increasing and where f is decreasing, and also the intervals where f is concave up and where f is concave down. Find the relative local maximum and minimum values of f and the points of inflexion of f. Use this information to sketch the graph of $y = f(x)$.

(a) Domain $y = f(x) = x^3 - 3x$ is $x \in \mathbb{R}$ $\therefore$ no VA

(b) y-int $f(0) = (0)^3 - 3(0) = 0$ $(0,0)$

x-int $0 = x^3 - 3x$
$= x(x^2 - 3) = x(x - \sqrt{3})(x + \sqrt{3})$ x-int $(0,0)$, $(\sqrt{3},0)$ $(-\sqrt{3},0)$

HA $\lim_{x \to \infty} f(x) = \infty$ $\therefore$ no HA

(c) $f'(x) = 3x^2 - 3$ ✓
$= 3(x^2 - 1) = 3(x-1)(x+1)$
$\underset{1}{\downarrow}$ $\underset{-1}{\downarrow}$ ✓

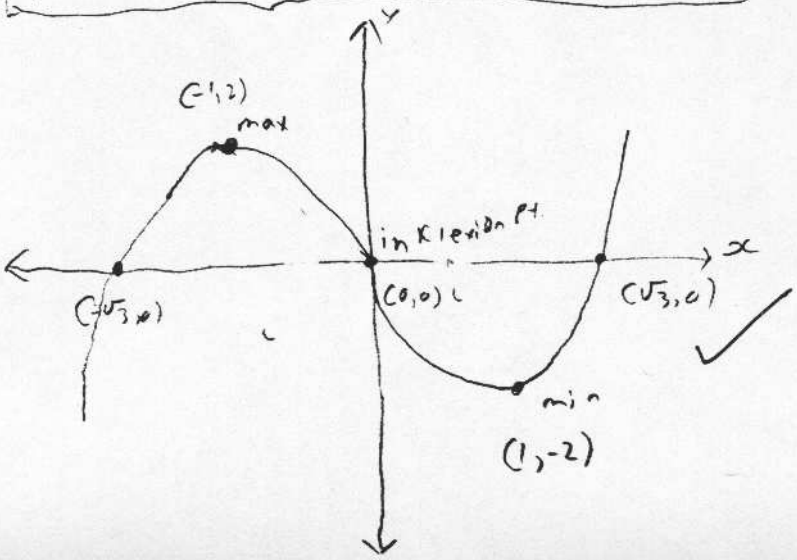
$(x-1)$
$(x+1)$
$+$ -1 $-$ 1 $+$

$f(x)$ inc $(-\infty, -1)$, $(1, \infty)$ ✓
$f(x)$ dec $(-1, 1)$
local max $f(-1) = 2$ $(-1, 2)$
local min $f(1) = -2$ $(1, -2)$ ✓

$f''(x) = 6x$ ✓
$\Rightarrow 0$ ✓

$(-\infty, 0)$ $f''(x) < 0$ then $f(x)$ ↺ concave down
$(0, \infty)$ $f''(x) > 0$ then $f(x)$ ↻ concave up ✓

inflexion pt at $f(0) = 0$ $(0,0)$ ✓



good!