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Math 110-004: Calculus I

Midterm exam # 1, October 18, 1999.

Name:

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1 - (40 pts.) Find the first and second derivatives of the following functions.

a) $f(x) = x^4 + 4x$

$f'(x) = 4x^3 + 4$

$f''(x) = 12x^2$

b) $V(x) = (x^4 + 4x)^5$

$V'(x) = 5(x^4 + 4x)^4 (4x^3 + 4)$

c) $H(x) = \sqrt{x^4 + 4x}$

d) $g(x) = \dfrac{1}{x^4 + 4x}$

2 - (15 pts.) In Regina, when a tennis ball is shot upwards with a velocity of 19.6 m/s, its height $H(t)$ (in meters) after t seconds is given by $H(t) = 19.6t - 4.9t^2$.

a) What is its average velocity between the times $t = 1$ and $t = 2$?

b) What is its average velocity between the times $t = 1$ and $t = 1.01$?

c) What does the quantity $\lim_{t \to 1} \frac{H(t) - H(1)}{t - 1}$ represent?

d) What is the instantaneous velocity of the tennis ball at the time $t = 1$?

15 (a) $v_{ave} = \frac{\Delta d}{\Delta t}$ ... $H(1) = 19.6(1) - 4.9(1)^2 = 14.7$ m

$H(2) = 19.6(2) - 4.9(2)^2 = 19.6$ m

(d) $H'(1) = 19.6$ ...

3 - (15 pts.) A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 20 cm. by 30 cm. by cutting out equal squares of side x at each corner and then folding up the sides.

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a) Express the volume $V(x)$ of the box as a function of x.

b) Which value of x maximizes the volume of the box?

30 cm

(a) $V(x) = (30 - 2x)(20 - 2x)x$

$V(x) = 4x^3 - 100x^2 + 600x$

4 - (15 pts.) The derivative of the function f is $f'(a) = 20a^3 - 4a$ and the equation of the tangent line to the graph of $y = f(x)$ at the point $a = 1$ is $y = 16x - 10$. Find $f(x)$.

$f(a) = \frac{20a^4}{4} - \frac{4a^2}{2} + c$

$f(a) = 5a^4 - 2a^2 + c$

$\therefore f(x) = 5x^4 - 2x^2 + 3$

$y = 16(1) - 10$

$y = 6$

$\therefore f(1) = 6 = 5(1)^4 - 2(1)^2 + c$

$6 = 3 + c$

$c = 3$

15

5 - (15 pts.) a) Evaluate $\int 2 - x^2 - x^4 dx$

b) Evaluate $\int_{-1}^{1} 2 - x^2 - x^4 dx$

c) Sketch both the inverted parabola $y = 1 - x^2$ (which looks like ∩) and the curve $y = x^4 - 1$ (which looks like ∪) on the same graph.

d) What is the area between the two curves?

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(a) $F(x) = 2x - \frac{x^3}{3} - \frac{x^5}{5} + C$

(b) $F(x) = 2x - \frac{x^3}{3} - \frac{x^5}{5} \Big|_{-1}^{1}$

$= \left[2 - \frac{1}{3} - \frac{1}{5}\right] - \left[-2 + \frac{1}{3} + \frac{1}{5}\right]$ [illegible]

(c)

$y = 1 - x^2$ [illegible]

$y = x^4 - 1$ [illegible]

$(0,-1)$ $(1,0)$ [illegible]

(d) $\int_{-1}^{1} 1 - x^2 - \ldots$ [illegible]

$= 2x - \frac{x^3}{3} - \frac{x^5}{5} \Big|_{-1}^{1}$ [illegible]