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Name: \_\_\_\_\_

1 - (10 pts.) Find the first and second derivatives of the following functions.

4/2  
1/1)

a)  $f(x) = x^4 + 4x$

$f'(x) = 4x^3 + 4$

$f''(x) = 12x^2$

1/2

b)  $V(x) = (x^4 + 4x)^5$

$V'(x) = 5(x^4 + 4x)^4 (4x^3 + 4)$

$V''(x) = 20(x^4 + 4x)^3 (4x^3 + 4)(12x^2 + 4) + 12x^3(4x^3 + 4)^2$

c)  $H(x) = \sqrt{x^4 + 4x} = (x^4 + 4x)^{1/2}$

$H'(x) = \frac{1}{2}(x^4 + 4x)^{-1/2} (4x^3 + 4)$

$H''(x) = \frac{1}{2} \left[ -\frac{1}{2}(x^4 + 4x)^{-3/2} (4x^3 + 4)^2 + (12x^2)(x^4 + 4x)^{-1/2} \right]$

d)  $g(x) = \frac{1}{x^4 + 4x} = (x^4 + 4x)^{-1}$

$g'(x) = -1(x^4 + 4x)^{-2} (4x^3 + 4)$

$g''(x) = -1 \left[ -2(x^4 + 4x)^{-3} (4x^3 + 4)^2 + (12x^2)(x^4 + 4x)^{-2} \right]$



2 - (15 pts.) In Regina, when a tennis ball is shot upwards with a velocity of 19.6 m/s, its height  $H(t)$  (in meters) after  $t$  seconds is given by  $H(t) = 19.6t - 4.9t^2$ .

- What is its average velocity between the times  $t = 1$  and  $t = 2$ ?
- What is its average velocity between the times  $t = 1$  and  $t = 1.01$ ?
- What does the quantity  $\lim_{t \rightarrow 1} \frac{H(t) - H(1)}{t - 1}$  represent?
- What is the instantaneous velocity of the tennis ball at the time  $t = 1$ ?

15 (a)  $v_{ave} = \frac{\Delta d}{\Delta t} = \frac{19.6(2) - 4.9(2)^2 - (19.6(1) - 4.9(1)^2)}{2 - 1} = \frac{14.7}{1} = 14.7 \text{ m/s}$

(b)  $\bar{v} = \frac{\Delta d}{\Delta t} = \frac{19.6(1.01) - 4.9(1.01)^2 - (19.6(1) - 4.9(1)^2)}{1.01 - 1} = \frac{16.701}{0.01} = 1670.1 \text{ m/s}$

$\bar{v} = 16.701 \text{ m/s}$

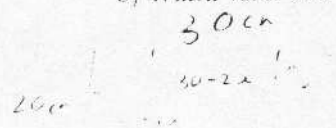
(c) It represents the instantaneous velocity of the ball at  $t = 1$ .

(d)  $v_{inst} = H'(t) = 19.6 - 9.8t$

$v_{inst} = H'(1) = 19.6 - 9.8(1) = 9.8 \text{ m/s}$

3 - (15 pts.) A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 20 cm. by 30 cm. by cutting out equal squares of side  $x$  at each corner and then folding up the sides.

- Express the volume  $V(x)$  of the box as a function of  $x$ .
- Which value of  $x$  maximizes the volume of the box?



(a)  $V(x) = (30-2x)(20-2x)x$   
 $= 600 - 100x + 4x^2$

$V(x) = 4x^3 - 100x^2 + 600x \quad 0 < x < 15$

(b)  $V'(x) = 12x^2 - 200x + 600$

Set  $V'(x) = 0$  to find critical points

$0 = 4(3x^2 - 50x + 150)$

$3x^2 - 50x + 150 = 0$

$x = \frac{50 \pm \sqrt{2500 - 1800}}{6}$

$x = \frac{50 \pm \sqrt{700}}{6}$

$x = 12.76 \text{ or } 3.91$

of volume

$x$  should be  $\approx 3.91$  to maximize the volume

4 - (15 pts.) The derivative of the function  $f$  is  $f'(a) = 20a^3 - 4a$  and the equation of the tangent line to the graph of  $y = f(x)$  at the point  $a = 1$  is  $y = 16x - 10$ . Find  $f(x)$ .

$$f'(a) = 20a^3 - 4a^2 + C$$

$$f'(a) = 5a^3 - 1a^2 + C$$

$$y = 16(1) - 10$$

$$y = 6$$

$$\int f'(x) = 5x^4 - 2x^2 + C$$

$$f(1) = 6 = 5(1)^4 - 2(1)^2 + C$$

$$6 = 3 + C$$

$$C = 3$$

15

5 - (15 pts.) a) Evaluate  $\int 2 - x^2 - x^4 dx$

b) Evaluate  $\int_{-1}^1 2 - x^2 - x^4 dx$

c) Sketch both the inverted parabola  $y = 1 - x^2$  (which looks like  $\cap$ ) and the curve  $y = x^4 - 1$  (which looks like  $\cup$ ) on the same graph.

d) What is the area between the two curves?

$$(a) F(x) = 2x - \frac{x^3}{3} - \frac{x^5}{5} + C$$

$$(b) F(x) = 2x - \frac{x^3}{3} - \frac{x^5}{5} \Big|_{-1}^1$$

$$= \left[ 2 - \frac{1}{3} - \frac{1}{5} \right] - \left[ -2 + \frac{1}{3} + \frac{1}{5} \right] = 2 - \frac{1}{3} - \frac{1}{5} - \left( -2 + \frac{1}{3} + \frac{1}{5} \right)$$

$$= 4 - \frac{2}{3} - \frac{2}{5} = \frac{60 - 10 - 8}{15} = \frac{42}{15}$$

(c)



$$y = 1 - x^2$$

$$y = x^4 - 1$$

Intersection points:  $(-1, 0)$ ,  $(1, 0)$

$$d) \int_{-1}^1 (1 - x^2 - (x^4 - 1)) dx$$

$$= \int_{-1}^1 (2 - x^2 - x^4) dx = 2x - \frac{x^3}{3} - \frac{x^5}{5} \Big|_{-1}^1$$

$$= \left[ 2 - \frac{1}{3} - \frac{1}{5} \right] - \left[ -2 + \frac{1}{3} + \frac{1}{5} \right]$$

$$= 4 - \frac{2}{3} - \frac{2}{5} = \frac{60 - 10 - 8}{15} = \frac{42}{15}$$