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Math 110-004: Calculus I

$\frac{98}{100}$

Midterm exam # 2, November 22, 1999.

Name:

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1 - (40 pts.) Find y'.

a) $y = \dfrac{\sin(x)}{x}$

$y' = \dfrac{\cos(x)\, x -}{x^2}$

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b) $y = \sqrt{\sec(x^2)} = (\sec(x^2))^{\frac{1}{2}}$

$y' = \frac{1}{2}(\sec(x^2))^{-\frac{1}{2}}\left(\tan(x^2)\sec(x^2)(2x)\right)$

$= \dfrac{\cancel{2}x\tan(x^2)\sec(x^2)}{\cancel{2}\sqrt{\sec(x^2)}} = x\tan(x^2)\sqrt{\sec(x^2)}$

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c) $y'' = \sec^2(x)$

$y' = \int \sec^2(x) = \tan(x) + C$

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d) $y\cos(x) + x\cos(y) = 1$

10 $y'\cos x - \sin(x)y + \cos(y) - \sin(y)y'x = 0$

$y'\cos x - \sin(y)y'x = \sin(x)y - \cos(y)$

$y'(\cos x - \sin(y)x) = \sin(x)y - \cos(y)$

$y' = \dfrac{\sin(x)y - \cos(y)}{\cos x - \sin(y)x}$

2 - (10 pts.) Rolle's theorem states that *if f is a function that is continuous on $[a, b]$ and differentiable on (a, b), and if $f(a) = f(b)$, then there is a number c in (a, b) such that $f'(c) = 0$.* For the function $f(x) = 3x + \sin(x)$ we have $f(0) = 0$. Use Rolle's theorem to show that there is no value of x except 0 such that $f(x) = 0$.

if $f(a) = f(b)$ ... $f'(c) = 0$

$f'(x) = 3 + \cos(x) \overset{?}{=} 0$

$\cos(x) \overset{?}{=} -3$ $\quad -1 \leq \cos(x) \leq 1$

$\cos(x) \neq -3$ [...]!

$3 + \cos(x) \neq 0 \quad f'(c) \neq 0$

So There is no value of x except 0 such that $f(x) = 0$

3 - (25 pts.) The area of a sphere of radius r is $A = 4\pi r^2$ and its circumference is $2\pi r$. For the following problem, **a.** find what quantities are given in the problem, **b.** find what is the unknown, **c.** write an equation that relates the quantities, **d.** finish solving the problem.

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Problem. A spherical snowball melts so that its surface area decreases at a rate of $1\,\text{cm}^2/\text{sec}$. Find the rate at which the circumference decreases when the radius is $10\,\text{cm}$.

(a) $A' = -1\,\text{cm}^2/s \quad r = 10\,\text{cm}$

(b) $C' = ?$

A' is rate the area is changing w.r.t. time; r is radius; C' is rate the circumference changes; r = 10 cm

(c) $C = 2\pi r \quad r = \dfrac{C}{2\pi}$

$A = 4\pi r^2$

$A = 4\pi\left(\dfrac{C^2}{4\pi^2}\right)$ simplify $\quad A = \dfrac{C^2}{\pi}$

(d) $A' = \dfrac{2CC'}{\pi}$

$A'\pi = 2CC'$

substitute for C

$C' = \dfrac{A'\pi}{4\pi r} = \dfrac{A'}{4r} = \dfrac{-1}{4\cdot 10} = \dfrac{-1}{40}\,\text{cm}/s$

The circumference decreases at a rate of $\frac{1}{40}$ cm/s

4 - (15 pts.) Evaluate the following limits.

a) $\lim_{x\to-\infty} \frac{\sqrt{3+4x^2}}{4+x}$ $\cdot \frac{\left(-\frac{1}{\sqrt{x^2}}\right)}{\left(-\frac{1}{x}\right)}$ $= -2$

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b) $\lim_{x\to\infty} \sqrt{x^2+x} - \sqrt{x^2+1}$

$= \lim_{x\to\infty} \frac{x^2+x-x^2-1}{\sqrt{x^2+x}+\sqrt{x^2+1}}$

$= \lim_{x\to\infty} \frac{1-\frac{1}{x}}{\sqrt{1+\frac{1}{x}}+\sqrt{1+\frac{1}{x^2}}} = \frac{1}{2}$

5 - (10 pts.) The equation $x^2 - xy + y^2 = 3$ represents a "rotated ellipse", that is, an ellipse whose axes are not parallel to the coordinate axes. Show that the points $(1, 2)$ and $(-1, -2)$ belong to the ellipse. Then use implicit differentiation to show that the tangent line to the ellipse is horizontal at these points.

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$2x - y - y'x + 2yy' = 0$

$2yy' - y'x = y - 2x$

$y'(2y - x) = y - 2x$

$y' = \frac{y-2x}{2y-x}$