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Midterm exam # 2, November 22, 1999.



Name:

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1 - (40 pts.) Find y'.

a) $y = \frac{\sin(x)}{x}$

$$y' = \frac{\cos(x) \cdot x - \sin(x)}{x^2}$$

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b) $y = \sqrt{\sec(x^2)}$, $y = (\sec(x^2))^{\frac{1}{2}}$
 $y' = \frac{1}{2} (\sec(x^2))^{\frac{1}{2}-1} (\tan(x^2) \sec(x^2) (2x))$

$$= \frac{x \tan(x^2) \sec(x^2)}{x \sqrt{\sec(x^2)}} = x \tan(x^2) \sqrt{\sec(x^2)}$$

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c) $y'' = \sec^2(x)$

$$y' = \int \sec^2(x) = \tan(x) + C$$

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d) $y \cos(x) + x \cos(y) = 1$

(1) $y' \cos(x) - \sin(x)y + \cos(y) - \sin(y)y' x = 0$

$$y' (\cos(x) - \sin(y)y' x) = \sin(x)y - \cos(y)$$

$$y' (\cos(x) - \sin(y)y' x) = \sin(x)y - \cos(y)$$

$$y' = \frac{\sin(x)y - \cos(y)}{\cos(x) - \sin(y)y' x}$$

$$\cos(x) - \sin(y)y' x$$

2 - (10 pts.) Rolle's theorem states that if f is a function that is continuous on $[a, b]$ and differentiable on (a, b) , and if $f(a) = f(b)$, then there is a number c in (a, b) such that $f'(c) = 0$. For the function $f(x) = 3x + \sin(x)$ we have $f(0) = 0$. Use Rolle's theorem to show that there is no value of x except 0 such that $f(x) = 0$.

if $f(a) = f(b) = 0$ then $f'(c) = 0$
 $f'(x) = 3 + \cos(x) = 0$
 $\cos(x) = -3$ [impossible]
 $3 + \cos(x) \neq 0$ for all $x \neq 0$
 10 There is no value of x except 0 such that $f(x) = 0$

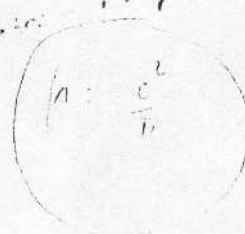
3 - (25 pts.) The area of a sphere of radius r is $A = 4\pi r^2$ and its circumference is $2\pi r$. For the following problem, a. find what quantities are given in the problem, b. find what is the unknown, c. write an equation that relates the quantities, d. finish solving the problem.

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Problem. A spherical snowball melts so that its surface area decreases at a rate of $1 \text{ cm}^2/\text{sec}$. Find the rate at which the circumference decreases when the radius is 10 cm .

(a) $A' = -1 \text{ cm}^2/\text{s}$ $r = 10 \text{ cm}$
 (b) $C' = ?$

Area is decreasing
 circumference is decreasing
 radius is constant

(c) $r = \frac{C}{2\pi}$
 $A = 4\pi r^2 = 4\pi \left(\frac{C}{2\pi}\right)^2 = \frac{C^2}{\pi}$


(d) $A' = 2C C'$
 $\frac{A'}{2C} = C'$

substitute for C
 $C' = \frac{A' \pi}{4r} = \frac{-1 \pi}{4(10)} = -\frac{\pi}{40} \text{ cm/s}$

The circumference decreases at a rate of $\frac{\pi}{40} \text{ cm/s}$



4 - (15 pts.) Evaluate the following limits.

a) $\lim_{x \rightarrow \infty} \frac{\sqrt{3+4x^2}}{4+x}$

Handwritten work: $\frac{\sqrt{\frac{3}{x^2} + 4}}{\frac{4}{x} + 1} = \frac{\sqrt{4}}{1} = 2$

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b) $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2+x} - \sqrt{x^2+1}}{1}$

Handwritten work:

$$= \lim_{x \rightarrow \infty} \frac{x^2+x - x^2-1}{(\sqrt{x^2+x} + \sqrt{x^2+1})} = \lim_{x \rightarrow \infty} \frac{x-1}{\sqrt{x^2+x} + \sqrt{x^2+1}} = \frac{1}{2}$$

5 - (10 pts.) The equation $x^2 - xy + y^2 = 3$ represents a "rotated ellipse", that is, an ellipse whose axes are not parallel to the coordinate axes. Show that the points (1,2) and (-1,-2) belong to the ellipse. Then use implicit differentiation to show that the tangent line to the ellipse is horizontal at these points.

Pt. (1,2) $x=1, y=2$ $(1)^2 - (1)(2) + (2)^2 = 3 = 3$
 ∴ (1,2) is a pt on the ellipse

Pt. (-1,-2) $x=-1, y=-2$ $(-1)^2 - (-1)(-2) + (-2)^2 = 3 = 3$
 ∴ (-1,-2) is a pt on the ellipse

$$2x - y - x^2 + 2y^2 = 0$$

$$2 + y' - 1^2 = y - 2x$$

$$y' = \frac{y-2x}{2+y}$$

$y' = 0$
 $0 = y - 2x$
 $y = 2x$

$y' = 0$
 $0 = y - 2x$
 $y = 2x$

∴ The tangent line at (1,2) and (-1,-2) are horizontal.